Abstracts:

March 4 (Tuesday)

Non-integrability of the n body problem

Andrzej Maciejewski (University of Zielona Góla, Poland)

We consider the classical planar n body problem. In the center of mass frame it admits the energy and the angular momentum first integrals. Integrability of the system restricted to common level of these first integrals is investigated. We show that if $n > 2$, the system is not integrable on all but one level corresponding to the zero value of the energy and angular momentum. In our proof we use differential Galois methods. We discuss also applicability of our approach to study integrability of charged n body problem.

This is a common work with Thierry Combot and Maria Przybylska.

. . . . . .

Perturbation approaches to non-existence of first integrals and commutative vector fields in continuous dynamical systems

Shoya Motonaga (Ritsumeikan University, Japan)

This talk concerns non-existence of first integrals and commutative vector fields in continuous dynamical systems from the view point of perturbation approaches. I describe recent developments of Poincaré's classical theorem and differential Galoisian obstructions for perturbed systems, which are closely related to the subharmonic/homoclinic Melnikov methods. As applications, I present non-integrability of periodically perturbed nonlinear oscillators and dissipative perturbed planar Hamiltonian systems. I also explain non-existence arguments on real analytic/meromorphic first integrals near periodic orbits of (not necessarily perturbed) systems, in which case a perturbation approach still works. This talk is partially based on a joint work with Kazuyuki Yagasaki (Kyoto University).

. . . . . .

Restricted integrability and Liouville Arnold integrability

Thierry Combot (University of Burgundy)

Consider an n degrees of freedom Hamiltonian H which admits m first integrals, and when restricted to M, a specific common level of these first integrals, admits $n-m$ first integrals. We will see that this notion of integrability does not coincide with Liouville Arnold integrability restricted to M. Still, some of the classical results hold. When $n-m=1$, for most systems, the motion is still quasi periodic on tori on a full measure set of the phase space, and the system is still integrable by quadratures. However, there is also a dense set on which the motion is not quasi periodic, and the angle coordinates are not smooth functions of the actions. When $n-m=2$, level sets are not always tori, and there are robust cases where the dynamic is almost nowhere quasi periodic. In such cases, we will prove there exist coordinates on a full measure set of the phase space in which the motion becomes parabolic. We will explore how the reduction of a Hamiltonian admitting

first integrals hides this fact, and that thus, Liouville Arnold integrability of the reduced Hamiltonian does not imply Liouville Arnold integrability of the original Hamiltonian.

. . . . . .

Algebraic aspects of real differential Galois theory

Teresa Crespo (University of Barcelona, Spain)

Differential Galois theory was established by Kolchin over differential fields with algebraically closed field of constants. In my talk I will present a joint work with Zbigniew Hajto and Marius van der Put on the existence of a real Picard-Vessiot field for linear differential systems defined over a real field with a real closed field of constants, as well as an unicity result [1]. The proof uses Deligne's work on Tannakian categories and a result of Serre on Galois cohomology. I will further discuss some features of Picard Vessiot fields and differential Galois groups in the real case [2].

[1] T. Crespo, Z. Hajto and M. van der Put, Real and p-adic Picard-Vessiot fields, Math. Ann. 365 (2016), no. 1-2, 93-103.
[2] T. Crespo and Z. Hajto, Picard-Vessiot extensions of real differential fields, SIGMA Symmetry Integrability Geom. Methods Appl. 15 (2019), Paper No. 100, 11 pp.

. . . . . .

Tame topology and non-integrability of real dynamical systems

Zbigniew Hajto (Jagiellonian University, Poland)

Following the works [1] and [2], I will discuss the topology of solutions of real nonintegrable systems appearing in problems of analytical mechanics. In the following part I will present some possible generalizations within the nonlinear differential Galois theory developed in the recent works of B. Malgrange (see [3] and the references therein).

[1] T. Crespo, Z. Hajto and M. van der Put, Real and p-adic Picard-Vessiot fields, Math. Ann. 365 (2016), no. 1-2, 93-103.
[2] Z. Hajto, R. Mohseni, Tame topology and non-integrability of dynamical systems, vol. 128, Banach Center Publications (2024), "GKLW-Workshop in Singularity Theory in memory of Stanisław Łojasiewicz", 95 -106 (2024).
[3] B. Malgrange, Deux lettres de Bernard Malgrange sur la théorie de Galois différentielle non-linéaire, Ann. Fac. Sci. Toulouse, Math. (6) 33, No. 1, 225-235 (2024)

. . . . . .

March 5 (Wednesday)

Semiclassical constant elasticity variance option pricing model: An analytical approach

Juan J. Morales-Ruiz (Universidad Politecnica de Madrid, Spain)

This talk will be devoted to obtain closed form solutions for the semiclassical (or WKB) approximation of the heat kernel propagator of the diffusion equation defined by the constant elasticity variance (CEV) option pricing model. One of the key points is that our calculations are based on the Van Vleck-Morette determinant instead of the Van Vleck determinant used by other authors. In fact, we compute this determinant in two different ways: by means of the solution of the classical Hamiltonian equations, and by solving the variational equations. Furthermore, the calculation reveals an exponential factor in the prefactor of the kernel, not considered in previous works. This is joint work with J.A. Capitán and J. Lope-Alba.

. . . . . .

Heteroclinic transition motions and nonintegrability for periodic perturbations of conservative systems

Kazuyuki Yagasaki (Kyoto University, Japan)

We consider time-periodic perturbations of conservative systems. The unperturbed systems are assumed to have two nonhyperbolic equilibria connected by a heteroclinic orbit on each level set of conservative quantities. These equilibria construct two normally hyperbolic invariant manifolds in the unperturbed phase space, and by invariant manifold theory there exist two normally hyperbolic, locally invariant manifolds in the perturbed phase space. We extend Melnikov's method to give a condition under which the stable and unstable manifolds of these locally invariant manifolds intersect transversely. Moreover, when the locally invariant manifolds consist of nonhyperbolic periodic orbits, we show that there can exist heteroclinic orbits connecting periodic orbits near the unperturbed equilibria on distinct level sets. This yields transition motions between neighborhoods of very distant periodic orbits, which are similar to Arnold diffusion for three or more degree of freedom Hamiltonian systems possessing a sequence of heteroclinic orbits to invariant tori, if there exists a sequence of heteroclinic orbits connecting periodic orbits successively. We also extend a recent result for time-periodic perturbations of single-degree-of-freedom systems and provide a sufficient condition for them to be not meromorphically integrable such that first integrals and commutative vector fields also depend on the small parameter meromorphically. We illustrate our theory for rotational motions of a periodically forced rigid body.

. . . . . .

Semi-analytic construction of global transfers between quasi-periodic orbits in the spatial R3BP

Amadeu Delshams (Universidad Politecnica de Catalunya, Spain)

Consider the spatial restricted three-body problem, as a model for the motion of a spacecraft relative to the Sun-Earth system. We focus on the dynamics near the equilibrium point L1, located between the Sun and the Earth. We show that we can transfer the spacecraft from a quasi-periodic orbit that is nearly planar relative to the ecliptic to a quasi-periodic orbit that has large out-of-plane amplitude, at zero energy cost. (In fact, the final orbit has the maximum out-of-plane amplitude that can be obtained through

the particular mechanism that we consider. Moreover, the transfer can be made through any prescribed sequence of quasi-periodic orbits in between).

. . . . . .

Hamiltonian instability in the elliptic Hill four-body problem: geometric method and numerical verification

Marian Gidea (Yeshiva University, USA)

We consider the planar elliptic restricted four-body problem (ER4BP), describing the dynamics of a massless body under the gravitational influence of three massive bodies (primaries) forming an equilateral central configuration, where each primary moves on an elliptic orbit about the common center of mass. We derive the elliptic Hill four-body problem (EH4BP), which is an approximation of the ER4BP describing the dynamics of the infinitesimal body in a neighborhood of the smaller body. The EH4BP can be written as a perturbation of the circular Hill four-body problem (CH4BP), with the eccentricity of the elliptic orbits being the small parameter.

We show that the EH4BP exhibits Hamiltonian instability, in the sense that there exist orbits of the infinitesimal body that undergo significant changes over time. Our mechanism of instability relies on the existence of two normally hyperbolic invariant manifolds (NHIM's), and on the corresponding homoclinic and heteroclinic connections.

A motivation for this work is the system consisting of Sun, Jupiter, the Trojan asteroid (624) Hektor, and its moonlet Skamandrios, which can be modeled by the EH4BP.

. . . . . .

Chaotic phenomena around L4 in the RPC3BP beyond the Routh mass ratio

Pau Martin (Universidad Politecnica de Catalunya, Spain)

The Restricted Planar Circular 3 Body (RPC3BP)problem is a simplified model of the 3BP in which one of the masses is zero, the other two, the primaries, describe circular orbits and the massless body evolves in the plane of the bodies with mass. It is well known that, in suitable rotating coordinates, this model has five equilibria, three of them colinear with the primaries, of saddle-center type, and the other two, commonly known as L_4 and L_5, forming an equilateral triangle with the primaries. The linear type of these last equilibria depends on the mass ratio: when this is smaller than the Routh value, they are of center-center type, becoming complex saddles for larger values, after a Hamiltonian-Hopf bifurcation.

It is well known that the rescaled normal form of the Hamiltonian at L_4 is formally integrable, when expanded in powers of the difference of the mass ration and the Routh value. Hence, deciding if the invariant manifolds of L_4 intersect transversely is a beyond all orders problem. Transversal intersection of the manifolds would give rise to Smale horseshoes in the problem, with the well known appearance of chaotic motions and periodic orbits of arbitrarily high period.

In this work we obtain an explicit expression for the distance between the invariant man-

ifolds of L_4. In particular, we prove that, if certain coefficient is different from 0, the invariant manifolds intersect transversely. This coefficient, often known as Stokes constant, depends on the full jet of the Hamiltonian. There is numerical evidence that it is not zero. To check that it is indeed different from 0 is work in progress.

. . . . . .

March 6 (Thursday)

Chaoticity of generic analytic convex billiards

Inma Baldoma (Universidad Politecnica de Catalunya, Spain)

This work deals with the abundance of analytic billiards having chaotic motions. The pioneer work by Zehnder's on planar twist maps in the 1970's was the first to provide a methodology for constructing analytic perturbations of maps in order to obtain transversality between the invariant manifolds of hyperbolic periodic orbits.

In this work, we prove that the set of analytic biliards with negative curvature having a transversal homoclinic orbit to periodic orbits of any rational rotation number is generic in the usual analytic topology. In other words, we prove that, for analytic biliards, the coexistence of chaotic dynamics with periodic orbit of any period is prevalent. We use the Aubrey-Mather theory to face with the transversality of periodic orbits away from the biliard's table boundary. This is a joint work with Anna Florio, Martin Leguil and Tere M-Seara.

. . . . . .

High-order persistence of resonant caustics in perturbed circular billiards

Rafael Ramirez-Ros (Universidad Politecnica de Catalunya, Spain)

We find necessary and sufficient conditions for high-order persistence of resonant caustics in perturbed circular billiards. The main technical tool is a high-order Melnikov theory based on the Bialy-Mironov generating function for convex billiards. A caustic is a curve such that any billiard trajectory, once tangent to the curve, stays tangent after every reflection. A convex caustic is p/q-resonant when all its tangent trajectories form closed polygons with q sides that make p turns around the caustic. We prove that any resonant caustic of the circular billiard with period q persist up to order $\lceil q/n \rceil - 1$ under any 'polynomial' perturbation of degree n of the circle. This is a join work with Comlan Edmond Koudjinan.

. . . . . .

Some quantitative aspects of action minimizing solutions

Kuo-Chang Chen (National Tsing Hua University, Taiwan)

Minimizing methods have been successfully applied to construct various types of periodic solutions for the n-body and n-center problems during the past two decades. Majority of relevant researches were endeavored to understand qualitative features such as existence,

uniqueness, and stability. In this talk we discuss a topic with relatively less attention — quantitative estimates for action values and mutual distances for action minimizing solutions. We will demonstrate some simple but nontrivial bounds. These estimates will facilitate numerical explorations to effectively locate and search new orbits.

. . . . . .

Variational construction of orbits realizing symbolic sequences in the planar Sitnikov problem

Mitsuru Shibayama (Kyoto University, Japan)

The Sitnikov problem is a special case of the three-body problem. The system is known to be chaotic and has been studied using symbolic dynamics. We study the limiting case of the Sitnikov problem, where the eccentricity of the massive particles tends to 1. Using the variational method, we demonstrate the existence of various kinds of solutions in the planar Sitnikov problem. We prove that for a given symbolic sequence, there exists an orbit realizing it and also establish the existence of periodic orbits. Moreover, we show the existence of heteroclinic orbits between periodic orbits in the planar Sitnikov problem. This is joint work with Yuika Kajihara and Guowei Yu.