6th Conference on Hamiltonian systems and related topics
– Three lectures in KAM theory by Prof. L. H. Eliasson and workshop –

Date: March 8 – 10, 2017
Place: Graduate School of Informatics, Kyoto University
Research Bldg. No.8, Main Campus
Lecture room 1 (1F 127) on March 8 and Lecture room 4 (3F 338) on March 9, 10.

Titles and Abstracts

Speaker: L. Hakan ELIASSON (Institut de Mathématiques de Jussieu-Paris, Rive Gauches, Université Paris Diderot)

Title: Three Lectures in KAM theory

1. Analytic Diophantine KAM-tori are never isolated

A KAM-torus (i.e. an invariant Lagrangian torus with a linear Diophantine flow) for a quasi-integrable Hamiltonian system is always accumulated by a set of KAM-tori of positive measure if the system is *non-degenerate*. What happens when the system is degenerate? We shall discuss this question based on a joint work with B. Fayad and R. Krikorian.

2. KAM for the beam equation

KAM has been applied successfully to prove the existence of quasi-periodic solutions for certain PDE's. In this lecture we shall discuss a recent result for the beam equation. (This is a joint work wit B. Grébert and S. Kuksin).

3. Reducibility and almost reducibility for quasi-periodic co-cycles

We shall discuss the notions of reducibility and almost reducibility of linear quasi-periodic co-cycles. We shall in particular consider the linear wave equation on a torus with a mass term and perturbed by a potential that depends quasi-periodically on time.

March 8

Speaker: Masayuki ASAOKA (Kyoto University)

Title: Real-analytic area preserving diffeomorphisms on the torus which exhibit super-exponential growth of the number of periodic points

Abstract: There are several results on abundance of smooth dynamical systems which exhibit super-exponential growth of the number of periodic points (Kaloshin, Bonatti-Diaz-Fisher, Asaoka-Shinohara-Turaev...). However, it has been unknown whether a real-analytic dynamical system can exhibit such fast growth or not. In this talk, we give an open subset of the set of real-analytic area-preserving diffeomorphisms on the torus in which diffeomorphisms exhibiting super-exponential growth of the number of periodic points are dense.

Speaker: Akira SHUDO (Tokyo Metropolitan University)

Title: Infinitely many stability islands and sticky dynamics in a piecewise linear map

Abstract: When regular and chaotic regions coexist in phase space, the orbits become sticky. It is known that survival and recurrence time distributions typically exhibit power law decay in such situations. Here we examine a two-dimensional piecewise linear map to explore the origin of stickiness. We particularly focus on 1) the case where the dynamics is almost hyperbolic except for a family of marginal periodic orbits and 2) the case where stability islands appear in a hierarchical way. In the latter case we find a way of partitioning the phase space which allows us to characterize hierarchical stability islands and to prove that stability islands exist infinitely many. We also discuss the origin of stickiness by comparing the conventional scenario based on the so-called Markov tree model proposed by Ott-Meiss with that found in the present piecewise linear map.

Speaker: Yutaka ISHII (Kyushu University)

Title: On parameter loci for the Hénon family

Abstract: The purpose of this talk is to investigate some geometric properties of the parameter locus of the Hénon family where the hyperbolic horseshoe dynamics breaks down. Among others we show that the hyperbolic horseshoe locus and the maximal entropy locus are completely characterized by a real analytic curve in the parameter space for the Hénon family, which extends the previous result of Bedford-Smillie in full generality. The proof of this result is based on the complexification of both the dynamical and the parameter spaces of the Hénon family together with computer assistance. This is joint work with Zin Arai (Hokkaido University).

Speaker: Hiroyoshi MITAKE (Hiroshima University)

Title: Selection problem for the discount Hamilton-Jacobi equation via the nonlinear adjoint method

Absract: We study the selection problem for the vanishing discount approximation of first-order Hamilton–Jacobi equations. It is known that the limit function of the vanishing discount procedure can be characterized in terms of Mather measures, which are invariant under the Euler-Lagrange flow. In this talk, we will introduce a way to construct these measures via the nonlinear adjoint method introduced by L. C. Evans, and establish the convergence. This way is versatile and we can deal with some of nonconvex Hamitlon-Jacobi equations and second-order Hamilton-Jacobi-Bellman equations. This is a joint work with Hung V. Tran (Univ. Wisconsin-Madison).

March 9

Speaker: Hiroki TAKAHASI (Keio University)

Title: Escape rate for Hénon-like maps at the first bifurcation

Abstract: We study the dynamics of a strongly dissipative Hénon-like diffeomorphism f at the first bifurcation parameter corresponding to the quadratic homoclinic or heteroclinic tangency inside the limit set. It is known that Lebesgue almost every initial point in the one-dimensional unstable manifold of the fixed saddle diverges to infinity under forward iteration. We show that the rate of escape from a given neighborhood of the non-wandering set is equal to

$$\sup_{\mu} \left\{ h_\mu(f) - \lambda^+(\mu) \right\}.$$

Here, $h_\mu(f)$ denotes the Kolmogorov-Sinaĭ entropy of (f, μ), $\lambda^+(\mu)$ the positive Lyapunov exponent of μ and the supremum is taken over all f-invariant ergodic Borel probability measures.

This formula supports the conjecture of Eckmann and Ruelle.

Speaker: Kohei SOGA (Keio University)

Title: Weak KAM theory for discount Hamilton-Jacobi equations and its application

Abstract: Weak KAM theory for discount Hamilton-Jacobi equations and corresponding discount Lagrangian/Hamiltonian dynamics is discussed. Then it is applied to error estimates for viscosity solutions in the vanishing discount process. The main feature is to introduce and investigate the family of α-limit points of minimizing curves, with some details in terms of minimizing measures. In error estimates, the family of α-limit points is effectively exploited with properties of the corresponding dynamical systems.

Spaker : Masahito OHTA (Tokyo University of Science)

Title: Strong instability of standing waves for nonlinear Schrödinger equations with harmonic potential

Abstract: We study strong instability of standing wave solutions $e^{i\omega t}\phi_\omega(x)$ for nonlinear Schrödinger equations with L^2-supercritical nonlinearity and a harmonic potential, where $\phi_\omega(x)$ is a ground state of the corresponding stationary problem. It was proved by Fukuizumi and Ohta (2003) that the standing wave is orbitally stable if ω is sufficiently small, and it is orbitally unstable if ω is sufficiently large. In this talk, under the same assumption as the latter case, we prove that the standing wave is strongly unstable, that is, there exist finite time blowup solutions with the initial data in arbitrarily small neighborhoods of the orbit of the standing wave.

March 10

Speaker: Masaya MAEDA (Chiba University)

Title: On long time dynamics of small solutions of nonlinear dispersive equations with potential

Absract: In this talk, we consider continuous and discrete nonlinear Schrd̈ingier equations (NLS) with potential. Under the assumption that Schrd̈ingier operator has several eigenvalues, it is well known that there exist small bound states (localized in space, periodic in time solutions). We will show that for the continuous NLS, even though the linear Schrd̈ingier equaiton has quasi-periodic (in time) solutions, there exists no quasi-periodic (in time) solutions and for discrete NLS, there can exists quasi-periodic (in time) solutions. Similar result holds for nonlinear Klein Gordon equations.

Speaker: Mitsuru SHIBAYAMA (Kyoto University)

Title: Non-integrability of the restricted n-body problem

Abstract: The Morales-Ramis theory has been applied to many Hamiltonian systems to prove the non-integrability. Especially Maciejewski and Przybylska proved that the three-body problem is not integrable by using the Morales-Ramis theory. In order to prove it, they studied the variational equations along the Euler and Lagrange homothetic solutions. In this talk, we will prove the non-integrability of the restricted n-body problem. It is more difficult, because there is no homothetic solution. We will focus on the singularity which the extended differential equations have, and then apply the Morales-Ramis theory to it.

Speaker: Hidekazu ITO (Kanazawa University)

Title: Normal form theory from the viewpoint of integrability near elliptic equilibria

Abstract: Normal form theory is a basic tool in the study of dynamical systems in the sense of writing a given system as a perturbed one from a system that can be solved explicitly. Therefore it is closely related to integrability of dynamical systems. In this talk, we will consider Liouville integrability of a general vector field in a neighborhood of an elliptic equilibrium point. In particular, we discuss *non-commutative* integrability of a vector field and show some results asserting equivalence between integrability and existence of a normalizing transformation to Poincaré-Dulac normal form.