ANALYTIC-LIOUVILLE-NONINTEGRABLE HAMILTONIAN SYSTEMS AND NONANALYTIC-INTEGRABILITY

MASAFUMI YOSHINO

ABSTRACT. In [2] Bolsinov and Taimanov studied a Hamiltonian system related with a geodesic flow which is smooth- Liouville-integrable and analytic-Liouville-nonintegrable. They showed that analytic nonintegrability is closely related with non Abelian property of a monodromy group of a return map of the flow. They also showed smooth integrability of an analytic-nonintegrable system by constructing concretely a first integral with essential singularity. Similar study was also made in [5].

In this talk, we study necessary conditions for analytic- Liouville- integrability of Hamiltonian systems with resonance dimension one or two. Our necessary condition for the analytic integrability may be seen as an analytic counterpart of Taimanov's monodromy condition. We then show the sectorial (smooth) integrability of an analytic-nonintegrable system. In proving this we use expansions in terms of functions involving essential singularity, which we call transseries. We note that such an expression was implicitly used by Taimanov in [2], while it was first introduced by Ecalle (cf. [4]) and used by several authors. (See also [3]). In discussing smooth integrability we also show a close relation between representation of first integrals by transseries and that by the Birkoff normal form theory for Hamiltonians with simple resonance (so-called S-normal form theory developped by Ito. (cf. [6])). Part of the results in this talk will be published in [1] as the jointwork with Werner Balser.

REFERENCES

[1] Balser, W. and Yoshino, M., Integrability of Hamiltonian systems and transseries expansions, *to be published in Math. Z.*

[2] Bolsinov, A.V. and Taimanov, I.A.: Integrable geodesic flows with positive topological entropy. Invent Math. **140** (3), 639-650 (2000).

[3] Costin, O., *Asymptotics and Borel summability*, Chapman Hall/CRC Monographs and Surveys in Pure and Applied Mathematics, 141. CRC Press, Boca Raton, FL, xiv+250 pp. ISBN: 978-1-4200-7031-6 (2009).

[4] Ecalle, J., Six lectures on transseries, analysable functions and the constructive proof of Dulac's conjecture, *Bifurcations and periodic orbits of vector fields (Montreal, PQ, 1992)*, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., 408, 75-184, Kluwer Acad. Publ., Dordrecht (1993).

[5] Gorni, G. and Zampieri, G.: Analytic-non-integrability of an integrable analytic Hamiltonian system. Differ. Geom. Appl. **22**, 287-296 (2005).

[6] Ito, H.: Integrability of Hamiltonian systems and Birkoff normal forms in the simple resonance case. Math. Ann. **292**, 411-444 (1992).

DEPARTMENT OF MATHEMATICS, GRADUATE SCHOOL OF SCIENCE, HIROSHIMA UNIVERSITY, 1-3-1 KAGAMIYAMA, HIGASHI-HIROSHIMA, HIROSHIMA 739-8526, JAPAN

E-mail address: yoshino@math.sci.hiroshima-u.ac.jp